Similitude Properties of High-Speed Laminar and Turbulent Boundary-Layer Incipient Separation

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A systematic unified comparative analysis is made of all the possible similitude rules pertaining to incipient separation viscous interaction for both laminar and turbulent flows. For supersonic and hypersonic turbulent flows a new result is derived that, unlike existing correlations, agrees with experimental data at high Reynolds numbers in that it predicts the incipient separation pressure to be independent of Reynolds number.

Nomenclature

C_f = skin friction coefficient, $2\tau_w/\rho_\infty U_\infty^2$
C_p = pressure coefficient, $2(p-p_\infty)/\rho_\infty U_\infty^2$
d_N = nose diameter
l = characteristic streamwise length
L = distance from nose to compression corner or incident shock impingement point
M = Mach number
p = static pressure
$\dot{q}_w$ = surface heat transfer rate
Re_∞ = Reynolds number based on freestream conditions and L
T = absolute static temperature
T_t = total temperature
T_{ref} = characteristic boundary-layer (Eckert) reference temperature
U_∞ = freesteam velocity
v_e = induced normal velocity at boundary-layer edge
x,y = streamwise and normal coordinates, respectively
β = $\sqrt{M^2-1}$
δ = boundary-layer thickness
$\bar{\chi}$ = viscous interaction parameter $(M_\infty^3/Re_\infty^{1/2})$
τ_w = shear stress at surface
ω = viscosity temperature-dependence exponent $(\mu \sim T^\omega)$

Subscripts

e = local edge of boundary layer
i = incipient separation
∞ = freestream conditions
o = basic undisturbed local flow ahead of compression region
w = wall conditions
s = separation point

I. Introduction

INCIPIENT separation of supersonic boundary-layer flows continues to be an important problem in aerodynamic studies of high-speed aircraft and missiles owing to deflected control surfaces, flares and protuberances, shock wave-boundary-layer interactions and exhaust plume-base flow interactions. Of particular importance is the turbulent case, especially at the very high Reynolds number, full-scale flight conditions of large military and civil transport aircraft and Space Shuttle vehicles. Although a considerable body of knowledge has been built up about incipient separation,[1,2] recent experiments[3-6] indicate that the existing similitude rules do not extrapolate to these large Reynolds number conditions and are unsuitable for scaling-up the results of wind-tunnel tests and aerodynamic design calculations.[2] Thus further basic experimental and theoretical studies are needed.

The present paper re-examines the similitude rules governing incipient separation of laminar and turbulent boundary layers at both supersonic and hypersonic speeds. In particular, we focus attention on high Reynolds number turbulent flows in the light of recent experimental evidence[3-6] that the incipient separation pressure becomes independent of Reynolds number. A new basic turbulent scaling result is developed that, unlike previous results, agrees with this high Reynolds number trend. A systematic study of all the possible basic similitudes governing the free viscous-inviscid interaction associated with incipiently-separating laminar or turbulent boundary layers is also given.

II. Similitude Properties of Incipient Separation

General Considerations

Consider a boundary-layer flow in an adverse pressure gradient due to some downstream cause (flap, step, or impinging shock, see Fig. 1). Since it has been observed experimentally that the upstream occurance of incipient separation is independent of the detailed cause and depends only on the properties of the incoming boundary layer and the *local* viscous-inviscid interaction,[7,8] it is possible to obtain useful similitude rules governing the flow. Place the origin at the distance L_c; then for the viscous and pressure gradient forces to be of the same order in the "free interaction" region

$$\frac{\partial \tau}{\partial y} \sim \frac{\mathrm{d}p_e}{\mathrm{d}x} \tag{1}$$

Note that while this equation is exact at the wall, its application to the entire boundary layer is a definitive order of magnitude postulate for the interaction process. In addition, the inviscid flow deflection is proportional to the displacement thickness growth

$$\frac{v_e}{U_\infty} \approx \frac{\mathrm{d}\delta^*}{\mathrm{d}x} \tag{2}$$

which induces a corresponding local pressure rise for $M_e > 1$ given by

$$p_e - p_o \approx \rho_\infty U_\infty^2 F(v_e/U_\infty) \tag{3}$$

where in linearized supersonic flow $F \approx v_e/\beta U_\infty$, whereas in small disturbance hypersonic flow assuming the tangent wedge approximation $F \approx v_e/M_\infty U_\infty$ and $F \simeq (v_e/U_\infty)^2$ for weak and strong interactions, respectively.

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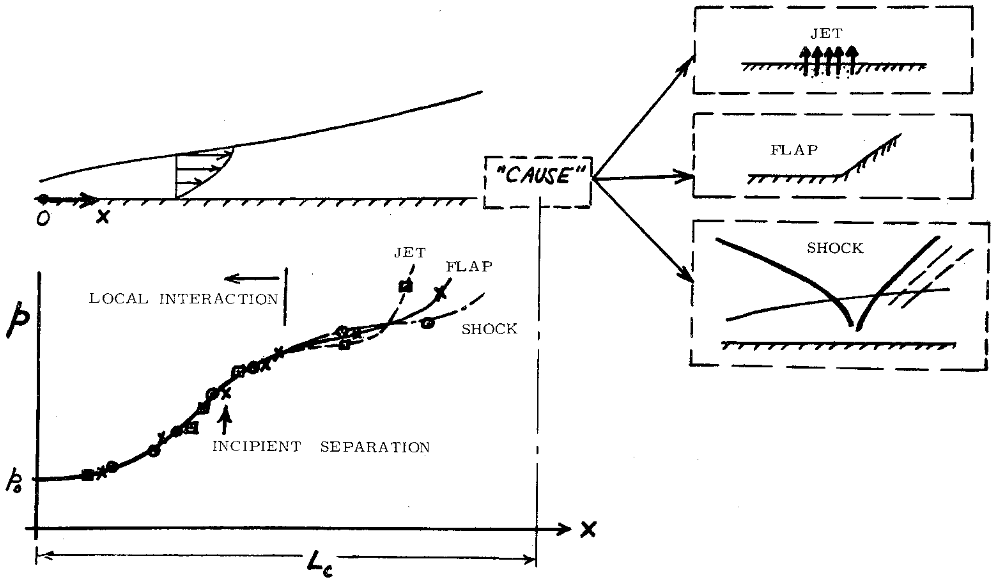

Fig. 1 Local interaction concept (schematic).

We characterize streamwise rates of change by a scale length l_i, the normal distance scale by δ^*, the characteristic maximum boundary-layer shear stress by some unknown value $\tau_{\max}$, and the streamwise pressure rise experienced over l_i by $p_e - p_o$. Then Eqs. (1-3) yield

$$p_e - p_o \sim (l_i/\delta^*)\tau_{\max} \tag{4}$$

$$p_e - p_o \sim \rho_\infty U_\infty^2 [F(\delta^*/l_i) - F_0(\delta^*/l_{io})] \tag{5}$$

It is seen that l_i and δ^* appear only in their ratio, which is a consequence of the local interaction assumption. Also, Eqs. (4) and (5) involve *three* unknowns l_i/δ^*, $\tau_{\max}$, and $p_e - p_o$ so that an additional assumption or equivalent information must be supplied. By varying this choice and that for F, one can study systematically within a simple framework *all* the various possible similitudes governing free interaction. In this regard, it is emphasized that Eqs. (4) and (5) are exactly the same basic premises that have been used to derive all existing correlations.

Supersonic Flows

Consider first the simplest case of linearized supersonic inviscid flow, neglecting viscous-inviscid interaction effects in the incoming flow by assuming $p_o \approx p_\infty$ and $F_o \approx 0$ (these effects are discussed in Ref. 2). Then with $F \approx \delta^*/\beta_\infty l_i$, Eqs. (4) and (5) yield

$$C_{p_i} \sim \frac{l_i}{\delta^*} C_{f\max} \tag{6}$$

$$C_{p_i} \sim (\delta^*/l_i\beta) \tag{7}$$

Different similitude rules can now be extracted depending on how one characterizes the interaction process.

For example, one could assume[9] that the interaction is weak enough that $C_{f\max} \approx C_{f0}$. Then Eqs. (6) and (7) yield $l_i/\delta^* \sim (\beta C_{f0})^{-1/2}$ (i.e., a well spread-out long range interaction) and $C_{p_i} \sim (C_{f0}/\beta)^{1/2}$. Hence for $\mu \sim T^\omega$ with $C_{f0} \sim [T_{\text{ref}}/T_\infty)^{1-\omega} Re_L]^{-1/2}$ or $[(T_{\text{ref}}/T_\infty)^{4-\omega} Re_L]^{-1/5}$ for laminar and turbulent flow, respectively, we obtain

$$\text{Laminar:} \quad C_{p_i} \sim \left[\left(\frac{T_{\text{ref}}}{T_\infty}\right)^{1-\omega} Re_L (M_\infty^2 - 1)\right]^{-1/4} \tag{8a}$$

$$\text{Turbulent:} \quad C_{p_i} \sim \left[\left(\frac{T_{\text{ref}}}{T_\infty}\right)^{4-\omega} Re_L\right]^{-1/10} (M_\infty^2 - 1)^{-1/4} \tag{8b}$$

where T_{ref}/T_∞ is of order unity at moderately supersonic speeds but of order $(\gamma-1)M_\infty^2$ in hypersonic flow. These relations predict a monotonic decrease in C_{p_i} with increasing Reynolds number; although the laminar scaling law (8a) generally agrees with experiment over the entire range of appropriate Reynolds (and Mach) numbers,[10] its turbulent counterpart (8b) correlates experimental data only at lower Reynolds numbers $Re_L < 10^7$.

Another approach has been suggested by Roshko and Thomke,[4] who again assume $C_{f\max} \approx C_{f0}$ but replace the "interaction" equation (5) with the short-range (strong interaction) hypothesis that $l_i \sim \delta^*$ for turbulent flows. This yields the linear relationship $C_{p_i} \sim C_{f_0}$ and hence

$$\text{Turbulent:} \quad C_{p_i} \sim \left[Re_L \left(\frac{T_{\text{ref}}}{T_\infty}\right)^{4-\omega}\right]^{-1/5} \tag{9}$$

Although this appears to correlate Kuehn's moderate Reynolds number turbulent separation data,[11] it too breaks down at higher Reynolds numbers. Since $l_i \sim \delta^*$ is inconsistant with the assumption that $C_{f\max}$ is on the order of its *undisturbed* value,[12] the range of validity of Eq. (9) is questionable; indeed, one would expect it to break down because it neglects the inviscid aspect of the flow embodied in Eq. (5) that becomes of dominant importance at high Reynolds numbers.

However, there is yet a *third* (and evidently heretofore overlooked) legitimate possibility whereby the strong interaction hypothesis $l_i \sim \delta^*$ is again invoked, but now with $C_{f\max}$ determined from the interaction Eq. (5), i.e., Eq. (5) is simply retained instead of being replaced by an *a priori* assumption. This yields the more self-consistent result $C_{f\max} \sim C_{pi}$ and

$$\text{Turbulent:} \quad C_{p_i} \sim (M_\infty^2 - 1)^{-1/2} \tag{10}$$

It is noteworthy that the same result is also obtained by invoking the alternative but equivalent turbulent flow hypothesis that $\tau_{\max} \sim \rho_\infty \epsilon U_\infty/\delta^*$ with $\epsilon \sim U_\infty \delta^*$. Although simple, Eq. (10) is significant because unlike the foregoing two scaling rules, it is completely independent of skin friction and hence of Reynolds number in agreement with experimental trends at moderate supersonic Mach numbers (Fig. 2). Moreover, Eq. (10) is also in qualitative agreement with Roshko and Thomke's data in showing an increase of p_i with Mach number.

This new strong interaction scaling for fully turbulent flow is substantiated by other experimental evidence as well.

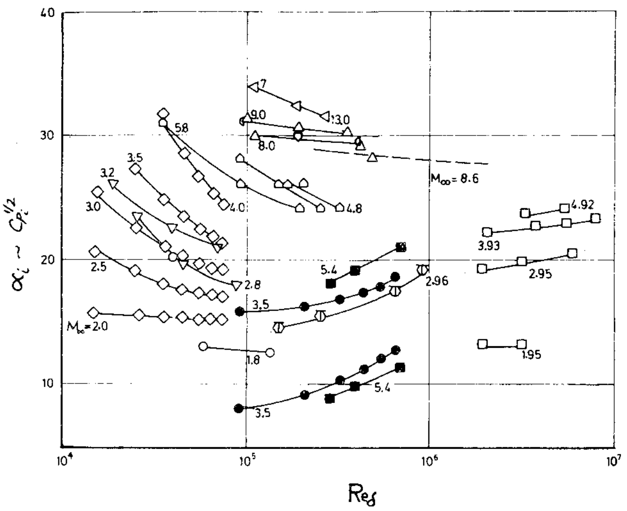

Fig. 2 Reynolds number effect on incipient separation of high-speed turbulent flows (various investigators [19]).

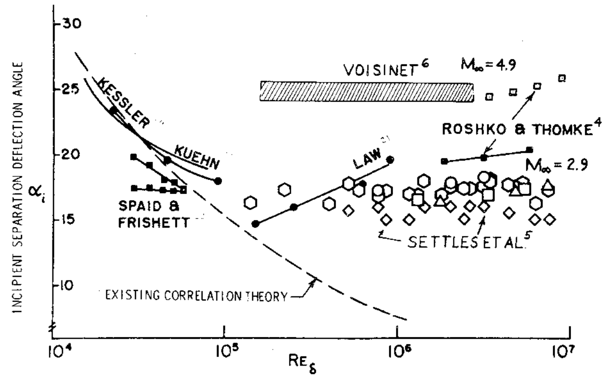

Fig. 3 Recent data showing Reynolds number independence of turbulent incipient separation.

Hammitt and Hight[13] observed that the incipient separation pressure was virtually independent of Reynolds number over the range $1.5 \leq Re_\delta \times 10^{-5} \leq 15$. More recently, Settles et al.[5] added further confirmation as shown in Fig. 3; they concluded that this behavior is the true state of affairs in purely turbulent incipiently-separating flow when it is free (as assumed herein) from Reynolds number-sensitive memory effects of upstream transition, especially due to artificial tripping.

Hypersonic Flows

Two types of local interaction can be distinguished according to whether the interaction is weak or strong.

Weak Interaction

Here, the hypersonic limit of the linearized theory $F \sim \delta^*/l_i M_\infty$ is used in Eq. (5):

1) Invoking the hypothesis $C_{f\max} \approx C_{f0}$, we find that $l_i/\delta_0^* \sim (C_{f0} M_\infty)^{-1/2}$ and $C_{p_i} \sim (C_{f0}/M_\infty)^{1/2}$ and hence with $T_{\text{ref}} \sim M_\infty^2 T_\infty$ that

Laminar: $$C_{p_i} \sim [M_\infty^{2(2-\omega)} Re_L]^{-1/4} \tag{11a}$$

Turbulent: $$C_{p_i} \sim [M_\infty^{13-2\omega} Re_L]^{-1/10} \tag{11b}$$

These results correspond to the hypersonic limit of the supersonic relations (8): they successfully correlate hypersonic incipient separation data in laminar and low-Reynolds-number turbulent flow[14] but fail at high Reynolds numbers.

2) A second similitude possibility, for interacting laminar flows, is of the type described by Hayes and Probstein[12]; they assume a long-range interaction distance $l_i \sim L$ with $\tau_{\max} \sim \mu_{\text{ref}} U_\infty/\delta_0$, $\delta_o \sim L Re_L^{-1/2}$. This leads to the familiar result $p_i/p_\infty \approx 1 + C M_\infty^{\omega-1} \bar{\chi}$ or

$$C_{p_i} \sim M_\infty^\omega Re_L^{-1/2} \sim M_\infty C_{f0} \tag{12}$$

with

$$l_i/\delta^* \sim (C_{f0} M_\infty^2)^{-1} \sim Re_L^{-1/2}/M_\infty^{\omega+1}$$

This linear dependence on the interaction parameter $\bar{\chi}$ (as opposed to the ⅔ power result discussed below) agrees with a variety of laminar hypersonic incipient data.[14,15]

3) The turbulent counterpart of the foregoing pertains to the long-range weak interaction assumptions $l_i \sim L$, $\delta^* \sim \delta_0^*$; it has also been given in a different context by Tang and Barnes.[16] Using the estimates

$$\delta_0 \sim (\mu_{\text{ref}}/\rho_{\text{ref}} U_\infty L)^{1/5}$$

$$C_{f0} \sim (T_\infty/T_{\text{ref}})(\rho_{\text{ref}} U_\infty L/\mu_{\text{ref}})^{1/5}$$

we obtain

$$p_i/p_\infty \approx 1 + (M_\infty^{2\omega+7}/Re_L)^{1/5}$$

and

$$c_{p_i} \sim (M_\infty^{3-2\omega} Re_L)^{-1/5} \sim M_\infty C_{f0} \tag{13}$$

with

$$l_i/\delta_0 \sim [Re_L/M_\infty^{2(\omega+1)}]^{1/5} \sim (C_{f0} \cdot M_\infty^2)^{-1}$$

As expected, this gives a far weaker Reynolds number dependence than the laminar case, but still incorrectly predicts that C_{p_i} monotonically decreases at very high Reynolds numbers. Indeed, *none* of these foregoing "weak interaction" similitudes is correct at $Re_L > 10^7$.

Strong Interaction

4) Assuming we now take $F \sim (\delta^*/l_i)^2$, $C_{f\max} \sim C_{f0}$ in this case; $l_i/\delta_0^* \sim C_{f0}^{-1/3}$, $C_{pi} \sim C_{f0}^{2/3}$ and hence with $T_{\text{ref}} \sim M_\infty^2 T_\infty$ that

Laminar:

$$C_{p_i} \sim \left[Re_L \left(\frac{T_{\text{ref}}}{T_\infty}\right)^{1-\omega}\right]^{-1/3} \sim M_\infty^{-2(1-\omega)/3} Re_L^{-1/3} \tag{14a}$$

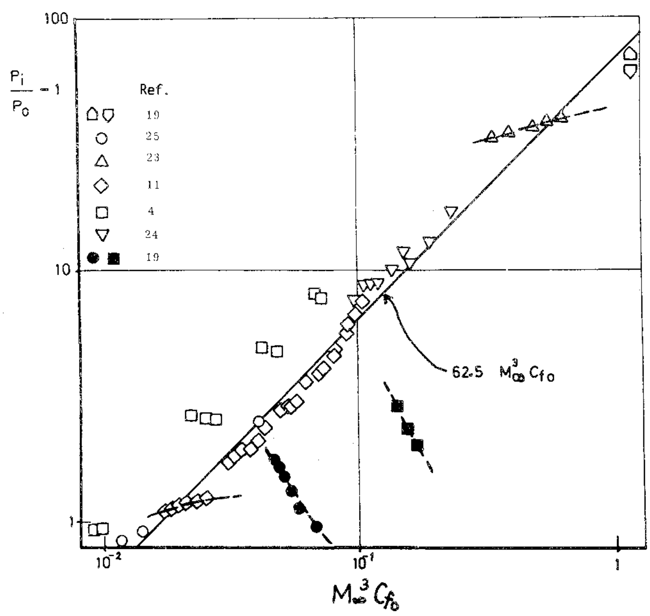

Fig. 4 Conventional correlation of turbulent incipient separation in hypersonic flow.

Table 1 Summary of similitude rules governing incipient separation ($\omega \approx 1$)

	Linearized supersonic ($C_{p_i} \sim \delta/\beta l_i$)		Small disturbance hypersonic: Weak interaction ($C_{p_i} \sim \delta/l_i M_\infty$)		Small disturbance hypersonic: Strong interaction ($C_{p_i} \sim \delta^2/l_i^2$)	
	l_i/δ	C_{p_i}	l_i/δ	C_{p_i}	l_i/δ	C_{p_i}
Long range						
Chapman-type ($C_f \sim C_{f_0}$); laminar or turbulent	$(\beta C_{f_0})^{-1/2}$	$(C_{f_0}/\beta)^{1/2}$	$(M_\infty C_{f_0})^{-1/2}$	$(C_{f_0}/M_\infty)^{1/2}$	$C_{f_0}{}^{-1/3}$	$C_{f_0}{}^{2/3}$
Hayes/Probstein ($l_i \sim L$); laminar	–	–	$(M_\infty^2 C_{f_0})^{-1}$	$M_\infty C_{f_0}$	$(Re_L/M_\infty^2)^{1/4}$	$M_\infty Re_L^{-1/2}$
Tang and Barnes	–	–	$(M_\infty^2 C_{f_0})^{-1}$	$M_\infty C_{f_0}$	–	–
Stollery and Bates ($l_i \sim L$); turbulent	–	–	–	–	$(Re_L/M_\infty^2)^{1/7}$	$(M_\infty^2/Re_L)^{2/7}$
Short range						
Roshko and Thomke ($l_i \sim \delta$, $C_f \sim C_{f_0}$); turbulent	0(1)	C_{f_0}	–	–	$0(1)$	$M_\infty^{-6/5} Re_L^{-1/5}$
Present theory	0(1)	$\beta^{-1/2}$	–	–	$0(1)$	$0(1)$

Turbulent:

$$C_{p_i} \sim \left[Re_L \left(\frac{T_{\rm ref}}{T_\infty} \right)^{4-\omega} \right]^{-2/15} \sim M_\infty^{-4(4-\omega)/15} Re_L^{-2/15} \qquad (14b)$$

both of which are quite different from their linearized supersonic counterparts (see Eq. 8). The laminar result (14a), which for $\omega = 1$ can be rewritten approximately as $p_i/p_\infty \sim \bar{\chi}^{2/3}$, was used by Album[17] to correlate hypersonic separation data for flared bodies. Equation (14b) for turbulent flow also predicts that C_{p_i} decreases monotonically with increasing Reynolds number, albeit at a somewhat steeper rate than the supersonic rule (Eq. 8b).

5) For laminar strong interactions, the approach described in Ref. 12 can be used: the long-range hypothesis $l_i \sim L$ is again adopted but *not* the assumptions $\delta \sim \delta_0$, $C_f \sim C_{f0}$; rather, one takes $\delta \sim (p_i/p_\infty)^{1/2} \delta_0$ and $C_f \sim (p_i/p_\infty)^{1/2} C_{f0}$. These lead to $p_i/p_\infty \sim M_\infty^{\omega-1} \bar{\chi}$ and

Laminar: $$C_{p_i} \sim M_\infty^{\omega} Re_L^{-1/2} \qquad (15)$$

with $l_i/\delta \sim (Re_L/M_\infty^2)^{1/4}$. The turbulent counterpart of this was discussed by Stollery and Bates[18]; taking $l_i \sim L$ with $\delta \sim (p_i/p_\infty)^{-1/5} \delta_0$ and $C_f \sim (p_i/p_\infty)^{4/5} C_{f0}$ reproduces their 2/7 power scaling law

Turbulent: $$C_{p_i} \sim (M_\infty^{2\omega}/Re_L)^{2/7} \qquad (16)$$

with

$$l_i/\delta \sim (Re_L/M_\infty^{2\omega})^{1/7}$$

6) Adopting the Roshko-Thomke short-range turbulent interaction approach by taking $C_{f_{\max}} \approx C_{f0}$ and $l_i \sim \delta$ yields the same Reynolds number dependence as in the supersonic case

Turbulent:

$$C_{p_i} \sim \left[Re_L \left(\frac{T_{\rm ref}}{T_\infty} \right)^{4-\omega} \right]^{-1/5} \sim M_\infty^{-2(4-\omega)/5} Re_L^{-1/5} \qquad (17)$$

where here the Mach-number dependence factor is seen to be strong. Equation (17) has the same limitations as its supersonic counterpart (9): it predicts a monotonic decrease with high Reynolds numbers where experiments indicate that C_{pi} becomes independent (or even slightly increases) with Re_L. This is illustrated in Fig. 4, where it is shown that while Eq. (17) does correlate some data, it also disagrees with the trends of several other experimental results obtained at higher Reynolds and Mach numbers.

7) Turning to the hypersonic version of the present turbulent strong-interaction analyses wherein we take $l_i \sim \delta$ but retain both Eqs. (4) and (5), we find $C_{f_{\max}} \sim C_{pi}$ with

Turbulent: $$C_{p_i} \sim 0(1) \qquad (18)$$

independent of Reynolds number as was its supersonic counterpart Eq. (10). This result is supported by the recent experiments of Voisinet[6] in the lower hypersonic Mach number range (Fig. 3). Moreover, Eq. (18) suggests that C_{pi} is independent of Mach number in hypersonic turbulent flow (implying $p_i \sim M_\infty^2$), in marked contrast to the implications of the other similitude rules. The trend of the meager high Mach number data for large Reynolds numbers displayed in Fig. 2 does suggest rather a weak Mach number effect, but much more experiment is needed to clarify this including the role of the significant $\partial p/\partial y$ effects under these conditions (see Ref. 2 for further discussion of this point).

III. Concluding Remarks

To provide a comparative overview of all the scaling laws governing high-speed incipient separation, a tabular summary of their predicted dependence of C_{pi} and l_i/δ on Reynolds and Mach number is given in Table 1. It shows that a remarkably large number of quite different similitude rules can be extracted from only a few simple basic equations. Moreover, it brings out the following important conclusions. First, all similitude rules based on assuming a long-range interaction predict a monotonic power-law decrease in C_{pi} with increasing Re_L. Consequently, although useful for laminar flows and in low Reynolds number turbulent flows ($Re_L < 10^7$) such theories inherently cannot account for the experimentally observed Reynolds number independence of turbulent incipient separation at higher Reynolds numbers. Second, in contrast, a short-range interaction hypothesis *can* explain the observed gross high Reynolds number behavior of turbulent flows *if* it is also consistently treated (as is done herein) as a strong interaction in the sense that $C_{f_{\max}}$ is *not* assumed equal *a priori* to the undisturbed value C_{f0}.

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References

[1] Chang, P.F., *Separation of Flow*, Pergamon Press, New York, 1970.

[2] Inger, G.R., "Similitude Properties of High Speed Laminar and Turbulent Boundary Layer Incipient Separation," VPI & SU Rept. Aero-037, Blacksburg, Va., Sept. 1975.

[3] Polak, A. and Kalivretenos, C.A., "Hypersonic Turbulent Boundary Layer Separations at High Reynolds Numbers," *AIAA Journal*, Vol. 6, Aug. 1970, pp. 954-95.

[4]Roshko, A. and Thomke, G.J., "Supersonic Turbulent Boundary Layer Interaction with a Compression Corner at Very High Reynolds Number," *ARL Viscous Interaction Symposium Proceedings* Dayton Univ. Press, Dayton, Ohio, 1969, pp. 109-138.

[5]Settles, G., Bogdanoff, S.M., and Vas. I.E., "Incipient Separation of a Supersonic Turbulent Boundary Layer at High Reynolds Numbers," *AIAA Journal*, Vol. 14, Jan. 1976, pp. 50-56.

[6]Voisient, R.L., "An Experimental Investigation of the Compressible Turbulent Boundary Layer Separation Induced by a Continuous Flow Compression," AGARD CP-168 (Flow Separation), Gottingen, F.R.G., Nov. 1975, pp. 19-1 to 19-11.

[7]Holden, M.S., "Experimental Studies of Separated Flows in Hypersonic Speeds. Part II: Two-Dimensional Wedge Separated Flow Studies," *AIAA Journal*, Vol. 4, May 1966, pp. 790-99.

[8]Holden, M.S., "Shock Wave-Turbulent Boundary-Layer Interaction in Hypersonic Flow," AIAA Paper 72-74, San Diego, Calif., 1972.

[9]Chapman, D.R., Kuehn, D.M., and Larson, H.K., "Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on the Effects of Transition," NACA Rept. 1353, 1955.

[10]Erdos, J. and Pallone, A., "Shock-Boundary Layer Interaction and Flow Separation," AVCO RAD TR-61-23, Wilmington, Mass., June 1961.

[11]Kuehn, D.M., "Experimental Investigation of the Pressure Rise Required for the Incipient Separation of Turbulent Boundary Layers in Two-Dimensional Supersonic Flow," NASA Memo 1-21-59, Feb. 1959.

[12]Hayes, W. and Probstein, R., *Hypersonic Flow Theory*, Academic Press, New York, 1959, Chap. IX.

[13]Hammitt, A.G. and Hight, S., "Scale Effects in Turbulent Shock Wave Boundary Layer Interactions," *Proceedings of the 6th Midwest Conference on Fluid Mechanics*, Sept. 1959, pp. 362-382.

[14]Needham, D.A., and Stollery, J.L., "Hypersonic Studies of Incipient Separation and Separated Flows," AGARD C.P. 4, 1966, p. 89-119.

[15]Bloy, A.W. and Georgeff, M.P., "The Hypersonic Laminar Boundary Layer Near Sharp Compression and Expansion Corners,"- *Journal of Fluid Mechanics*, Vol. 63, Pt. 3, 1974, pp. 431-447.

[16]Tang, H. and Barnes, J., "Strong and Weak Interaction Parameters for Turbulent Flow," *AIAA Journal*, Vol. 4, Oct. 1966, pp. 1838-1840.

[17]Album, H.A., "Hypersonic Laminar Separation on a Cone with Slip Flow," *AIAA Journal*, Vol. 7, Nov. 1969, pp. 2057-63.

[18]Stollery, J.L. and Bates, L., "Turbulent Hypersonic Viscous Interaction," *Journal of Fluid Mechanics*, Vol. 63, Pt. 1, 1974, pp. 145-157.

[19]Apfels, C. and Richards, B. R., "Incipient Separation of a Compressible Turbulent Boundary Layer," Paper 21, AGARD Conference on Flow Separation, CPP-168, May 1975.

[20]Kessler, W. C., "Supersonic Turbulent Boundary Layer Interaction with an Expansion Ramp and Compression Corner," McDonnell-Douglas Astronautics Corp., St. Louis, Mo., Rept. MDC EO-264, 1970.

[21]Law, H.C., "Supersonic Shock and Waves – Turbulent Boundary Layer Interactions," *AIAA Journal*, Vol. 14, June 1976, pp. 730-734.

[22]Spaid, F.J. and Frischnett, J.L., "Incipient Separation of a Supersonic Turbulent Boundary Layer Including Effects of Heat Transfer," *AIAA Journal*, Vol. 10, July 1972, pp. 915-923.

[23]Elfstrom, G.M., "Turbulent Separation in Hypersonic Flow," Imperial College Dept. of Aeronautics Report 11-16, London, Sept. 1971.

[24]Sterrett, J.R. and Emery, J.C., "Experimental Separation Studies for Two-Dimensional Wedges and Curved Surfaces at Mach Numbers of 4.8 to 6.2," NASA TN D-1014, Feb. 1962.

[25]Drougge, G., "An Experimental Inverstigation of the Influence of Strong Adverse Pressure Gradients on Turbulent Boundary Layers at Supersonic Speeds," Aeronautical Research Institute of Sweden Report FFA 47, Stockholm, 1953.